Grover algorithm (Nielsen Chuang, chapter 6)

search in unsorted database

e.g search for number telephonebook

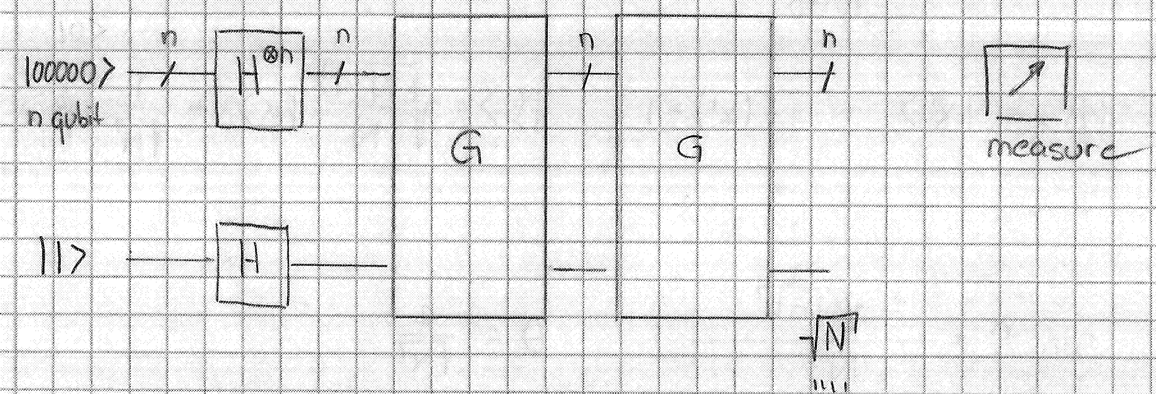
$f: \{\text{index}\} \longrightarrow \{\text{not found, found}\}$

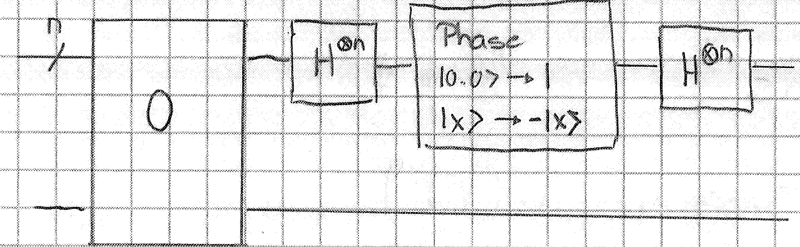
$f: \{0..00, 0..01, ... 111..\} \longrightarrow \{0, 1\}$

$N = 2^n$

$|x\rangle|y\rangle \xrightarrow{O} |x\rangle|y \oplus f(x)\rangle$

$|x\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} \xrightarrow{O} (-1)^{f(x)} |x\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}$





$$Ph: \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & \ddots \end{bmatrix} = 2|0\rangle\langle 0| - \mathbb{1}$$

$$H^{\otimes n}(2|0\rangle\langle 0| - \mathbb{1})H^{\otimes n} = 2|\xi\rangle\langle\xi| - \mathbb{1}$$

$$|\xi\rangle = H^{\otimes n}|0\rangle = \frac{1}{\sqrt{2^n}}\sum |x\rangle$$

$$= \frac{1}{\sqrt{2^n}}\left(|0..0\rangle + |0...01\rangle \ldots |1..1\rangle\right)$$

$2|\xi\rangle\langle\xi| - \mathbb{1}$

$$\sum_k \alpha_k |k\rangle \longrightarrow \sum_k \left(2\langle\alpha\rangle - \alpha_k\right)|k\rangle \quad \text{where} \quad \langle\alpha\rangle = \frac{1}{N}\sum_k \alpha_k$$

rotation about the mean

$$G = \left(2|\xi\rangle\langle\xi| - \mathbb{1}\right) O$$

geometric visualization

$$|\alpha\rangle = \frac{1}{\sqrt{N-1}} \sum_{\substack{x \\ f(x)=0}} |x\rangle$$

$$|\beta\rangle = |x'\rangle \qquad f(x') = 1 \qquad |\xi\rangle = \sqrt{\frac{N-1}{N}}\,|\alpha\rangle + \frac{1}{\sqrt{N}}|\beta\rangle$$

$$\sin\theta = \frac{2\sqrt{N-1}}{N} \qquad \theta \approx \frac{2}{\sqrt{N}}$$

$\sqrt{N}$ iterations

quadratic speedup $O(\sqrt{N})$

probabilistic

optimal

- quantum Fourier

  Deutsch-Josz

  Shor

  → Factoring

  exponential speedup

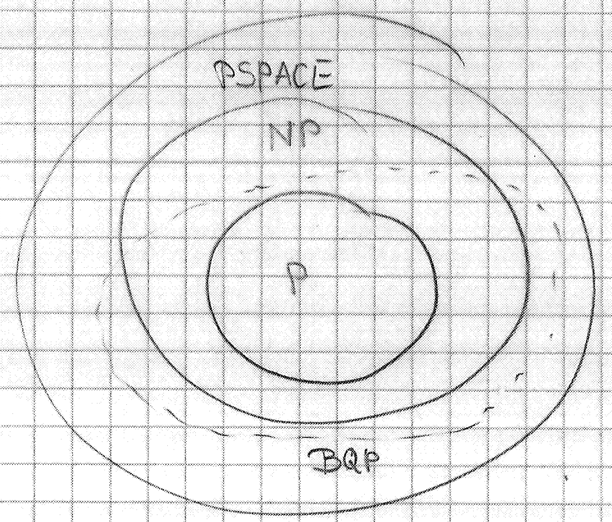
$N\log(N)$: $n2^n$

$\log(N)^2$ n^2

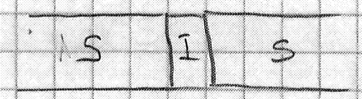
- quantum search

  $O(\sqrt{N})$

- quantum simulation

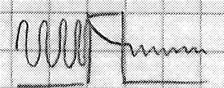


Josephson Effect



few Å

Al Al_2O_3 Al

tunnel effect: 

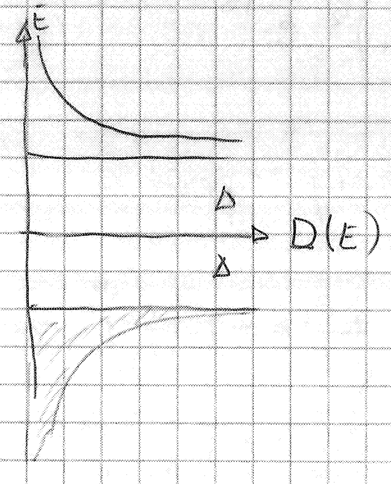
$e^{-\kappa x}$ $\kappa = \frac{1}{\hbar}\sqrt{2m(V_0 - E)}$

superconductivity

Bardeen Cooper Schrieffer

Cooper pairs $2e$ $(\vec{k}; \vec{k})$

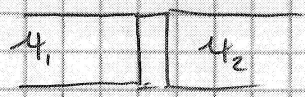
condensate $\psi = \sqrt{n}\, e^{i\theta}$



superconductivity class

Zehetmayer / Weber

DC Josephson Effect



$$i\hbar \frac{\partial}{\partial t}\psi_1 = \hbar T \psi_2 \qquad i\hbar \frac{\partial}{\partial t}\psi_2 = \hbar T \psi_1$$

$$\psi_1 = \sqrt{n_1}\, e^{i\theta_1} \qquad \psi_2 = \sqrt{n_2}\, e^{i\theta_2} \qquad n_i, \theta_i \in \mathbb{R}$$

$$\frac{\partial}{\partial t}\psi_1 = \frac{1}{2} n_1^{-1/2} e^{i\theta_1} \frac{\partial n_1}{\partial t} + i\, n_1^{1/2} e^{i\theta_1} \frac{\partial \theta_1}{\partial t} = -iT\psi_2$$

$$\frac{\partial}{\partial t}\psi_2 = \frac{1}{2} n_2^{-1/2} e^{i\theta_2} \frac{\partial n_2}{\partial t} + i\, n_2^{1/2} e^{i\theta_2} \frac{\partial \theta_2}{\partial t} = -iT\psi_1$$

multiply by $n_1^{1/2} e^{-i\theta_1}$

$$\frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -iT(n_1 n_2)^{1/2} e^{i\delta} \qquad \delta = \theta_2 - \theta_1$$

$$\frac{1}{2}\frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -iT(n_1 n_2)^{1/2} e^{-i\delta}$$

real part $\qquad\qquad$ imaginary part

$$\frac{\partial n_1}{\partial t} = 2T(n_1 n_2)^{1/2}\sin\delta \qquad \frac{\partial \theta_1}{\partial t} = -T\left(\frac{n_2}{n_1}\right)^{1/2}\cos\delta$$

$$\frac{\partial n_2}{\partial t} = -2T(n_1 n_2)^{1/2}\sin\delta \qquad \frac{\partial \theta_2}{\partial t} = -T\left(\frac{n_1}{n_2}\right)^{1/2}\cos\delta$$

equal superconductors $\quad n_1 \cong n_2$

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t} \qquad \frac{\partial}{\partial t}\delta = 0$$

$$\frac{\partial n_2}{\partial t} = -\frac{\partial n_1}{\partial t}$$

$$I = 2e\frac{\partial n_1}{\partial t}$$

$$\boxed{I = I_0 \sin(\delta) = I_0 \sin(\theta_2 - \theta_1)}$$

DC Josephson effect

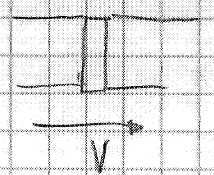
DC supercurrent without voltage

I_0 junction property

$\propto$ exp. thickness

$\propto A$

AC Josephson effect



$$i\hbar \frac{\partial}{\partial t}\psi_1 = \hbar T \psi_2 - eV\psi_1$$

$$i\hbar \frac{\partial}{\partial t}\psi_2 = \hbar T \psi_1 + eV\psi_2$$

$$\frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = \frac{i}{\hbar} eVn_1 - iT(n_1 n_2)^{1/2} e^{i\delta}$$

$$\frac{\partial n_1}{\partial t} = 2T(n_1 n_2)^{1/2} \sin(\delta)$$

$$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - T(n_1 n_2)^{1/2} \cos\delta$$

$$\boxed{\frac{\partial}{\partial t}(\theta_2 - \theta_1) = \frac{\partial}{\partial t}\delta = -2eV/\hbar}$$

AC Josephson effect

$$I = I_0 \sin\left(\delta(0) - \frac{2eV}{\hbar} t\right)$$

cooper pairs

$$\omega = \frac{2eV}{\hbar} \qquad \hbar\omega = 2eV$$

Voltage standard

$$\nu = \frac{2 \cdot 1{,}6 \cdot 10^{-19} \cdot 10^{-6}}{6{,}6 \cdot 10^{-34}} \approx \frac{1}{2} \cdot 10^{9} \approx 500\ \text{MHz}$$

$$V = \frac{\phi_0}{2\pi}\dot{\delta} \qquad \phi_0 = \frac{h}{2e}$$ superconducting flux quantum

$$= 2 \cdot 10^{-15}\ \text{Wb} = 20\ \text{Gauss}\ (\mu m)^2$$

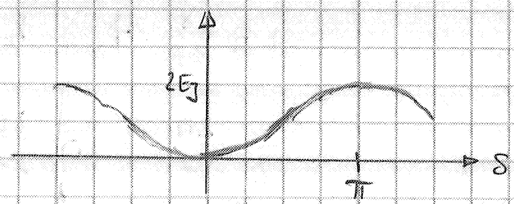
$$\text{Wb} = \text{Tesla}\ m^2$$

Josephson energy

$$E(\delta) = \int_0^t I V \, d\tau = \frac{I_0 \phi_0}{2\pi} \int_0^t \sin(\gamma) \frac{d\gamma}{d\tau} \, d\tau$$

$$= \frac{I_0 \phi_0}{2\pi} \int_0^\delta \sin(\gamma) \, d\gamma = \frac{I_0 \phi_0}{2\pi} \left(1 - \cos(\delta)\right)$$

$$\boxed{E = E_J \left(1 - \cos(\delta)\right) \qquad E_J = \frac{I_0 \phi_0}{2\pi}}$$



Josephson inductance

$$L = \frac{V}{\dot{I}} = \frac{\phi_0/2\pi \ \dot{\delta}}{I_0 \frac{d}{dt}(\sin(\delta))} = \frac{\phi_0}{2\pi I_0} \frac{\frac{d\delta}{dt}}{\cos(\delta)\frac{d\delta}{dt}}$$

$$\boxed{L = \frac{\phi_0}{2\pi I_0} \frac{1}{\cos(\delta)}}$$